HÖGSKOLAN
I SKÖVDE

Institutionen för ingenjörsvetenskap

TENTAMEN

Kurs	Diskret matematik (Discrete Mathematics)
Delkurs	Salstentamen (Written-Exam)
Kurskod	MA126 G1N
Högskolepoäng för tentamen	5,0
Datum	2025-05-16
Skrivtid	14.15-19.30

Ansvarig lärare Yohannes Aklilu

Berörda lärare

Hjälpmedel/bilagor Studentens miniräknare, högskolans miniräknare, Befogat formelblad

Övrigt

Anvisningar

- ☐ Ta nytt blad för varje lärare
- ☒ Ta nytt blad för varje ny fråga
- ☒ Skriv endast på en sida av papperet.
- ☒ Skriv namn och personnummer på samtliga inlämnade blad.
- ☒ Numrera lösbladen löpande.
- ☒ Använd inte röd penna.
- ☒ Markera med kryss på omslaget vilka uppgifter som är lösta.

Poänggränser

U-betyg(Fail grade): Not fulfilling the passing criterion.
G-betyg(Pass grade): At least 18 points.
VG-betyg (Pass with distinction grade): At least 28 points.

Skrivningsresultat bör offentliggöras inom 18 arbetsdagar

Lycka till!

Antal sidor totalt

Examination Instructions

The exam is assessed with a grade of *Pass with distinction* (VG), *Pass* (G) or *Fail* (U) based on how well your solutions demonstrate that the grading criteria for the course objectives have been met. Each task can give up to 6 points, a total of 36. For G-grade, a total of at least 18 points is required, for VG-grade at least 28.

To pass the examination with G-grade: Your answer to the tasks must be concise, but sufficiently detailed and formulated so that the line of thought can be easily followed. Some degree of calculation error can be acceptable, as long as the layout and motivation is correct. An answer, for example, with out any motivation, however, will not be accepted. Numerical values can be entered as expressions, suitably simplified, where root expressions, logarithms and exponential expressions can be included in addition to *pure numbers*, if needed.

In order to pass the exam with distinction (VG-grade): Your answer is required to be well-grounded and followed well-constructed mathematical reasoning that leads to a correct answer or conclusion. The answers must be well formulated and analysed, and draw relevant conclusions about the nature of the solutions in a logical and consistent manner.

The following course goals will be assessed on this examination:

- explain the central concepts and methods in discrete mathematics treated in the course,
- show good familiarity with integers and modular arithmetic, and in particular show some familiarity with the Euclidian algorithm, Fermat's little theorem, Euler's theorem and solve problems where they may be used,
- describe different graph theoretical relations, algorithms and their applications, e.g. graph searching, Kruskal's and Dijkstra's algorithms,
- analyze discrete mathematical structures and determine their characteristics using mathematical reasoning; typical examples of such structures are relations, graphs and trees,
- identify problems which can be solved by methods from discrete mathematics, and choose suitable methods and apply them in a structured way.

Good Luck!
YA

Examination tasks

Write your solutions on separate paper. Use new sheet for each task.

1. Let the universal $\mathbb{U}$ set be the set of all positive integers ≤ 30. Given the sets

$$A = \{x \in \mathbb{U} \mid x \text{ is prime}\} \quad \textit{and} \quad B = \{y \in \mathbb{U} \mid y \text{ is odd}\}.$$

(2p/task)

   (a) Are the following statements True or False? Justify your answer.

      i. The set $(B \setminus A)$ is empty set.

      ii. $(\forall x \in A)(\gcd(x, 30) = 1)$.

   (b) Choose any element $y \in B$ such that $y > 1$ and find its inverse modulo 30, or justify why your chosen number is not invertible.

   (c) Define an injective (one-to-one) function $f : A \to B$, or justify why it is not possible to define such a function.

2. (3p/task)

   (a) Let p, q and r be logical statements. Construct the truth table for the logical expression

$$((\neg r \vee p) \to q)) \leftrightarrow (\neg p \to (r \wedge \neg q)).$$

   (b) Given the following boolean matrices:

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} \quad \textit{and} \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

   Compute the matrices $A^t \odot B$ and $B \vee A$, $A \wedge B$ or justify why it is not possible to perform these operations.

3. Given a logical expression whose inorder traverse is: (2p/task)

$$p \quad \wedge \quad q \quad \vee \quad \neg \quad r \quad \leftrightarrow \quad p \quad \wedge \quad \neg \quad r \quad \to \quad p$$

   (a) Represent the logical expressions with a binary tree.

   (b) Use Depth-First Search to traverse the binary tree.

(c) Find the truth value of the logical expression if p and q are both 0 (False) and r is 1 (True).

4. (a) Solve the linear Diophintine equation $35x + 60y = 45$. (2p)

   (b) Convert the integer $(A2E)_{HEX}$ given in hexadecimal form to binary form. (2p)

   (c) If $a + 3 \cong 5 \pmod{11}$ and $2b - 1 \cong 8 \pmod{11}$, then find the smallest non-negative integer x such that $x \cong (3a + 6b) \pmod{11}$. (2p)

5. (a) A graph G has the nodes $V = \{a, b, c, d, e, f, g, h, i\}$ and adjacency matrix, where the natural ordering of alphabets are used to construct the matrix,

$$G_M = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

(4p)

   i. Justify whether the graph is directed or connected.

   ii. Justify if the graph has a Euler circuit.

   (b) Define the concept Hamiltonian path. Construct an example of a graph with 8 nodes that has a Hamiltonian path. (2p)

6. (a) Given the set $A = \{1, 2, 3, 4, 5, 6, 7\}$. (2pts/task)

   i. Construct the matrix representation of a relation R defined on the set A that is symmetric, but not reflexive.

   ii. Let

$$\mathcal{C} = \{\{1, 4, 5\}, \{2, 6\}, \{3\}, \{7\}\}$$

   be partition of A. Define an equivalence relation S on A whose equivalence classes are the partition $\mathcal{C}$.

   (b) Let a, b and c be positive integers. Prove: If a divides bc and $\gcd(a, c) = 1$, then a divides the integer b.

Discrete Mathematics, MA126G, 7,5hp
Formula Sheet

Logic

p	$\neg p$
1	0
0	1

p	q	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
1	1	1	1	1	1
1	0	0	1	0	0
0	1	0	1	1	0
0	0	0	0	1	1

Some set theoretic and logical relations or properties

	Set theoritical rules	**Logical Equivalences**
Dubble Complement	$(A^c)^c = A^{cc} = A$	$\neg\neg p \Leftrightarrow p$
Complement/negation	$A \cap A^c = \emptyset$	$p \wedge \neg p \Leftrightarrow 0$
	$A \cup A^c = U$	$p \vee \neg p \Leftrightarrow 1$
Idempotent	$A \cap A = A$	$p \wedge p \Leftrightarrow p$
	$A \cup A = A$	$p \vee p \Leftrightarrow p$
Commutative laws	$A \cap B = B \cap A$	$p \wedge q \Leftrightarrow q \wedge p$
	$A \cup B = B \cup A$	$p \vee q \Leftrightarrow q \vee p$
Associative laws	$A \cap (B \cap C) = (A \cap B) \cap C$	$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$
	$A \cup (B \cup C) = (A \cup B) \cup C$	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$
Distributive laws	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
DeMorgan rules	$(A \cup B)^c = A^c \cap B^c$	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
	$(A \cap B)^c = A^c \cup B^c$	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
	$A \backslash B = A \cap B^c$	$p \to q \Leftrightarrow \neg p \vee q$

Definitions and some operations on logic and set theory

Power set	$2^A = \{B \mid B \subseteq A\}$	
Cartesian product	$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$	
Quantifiers	$\exists x$	There exists x
	$\forall x$	For all x
	$\neg(\forall x P(x)) \Leftrightarrow \exists x(\neg P(x))$	
	$\neg(\exists x P(x)) \Leftrightarrow \forall x(\neg P(x))$	

Matrices

Given matrices (or Boolean matrices) $A = (a_{ij})$ and $B = (b_{ij})$. We have the following operations as long as compatibility criterions are satisfied:

Matrix addition/subtraction	$A \pm B = (a_{ij} + b_{ij})$
Matrix multiplication	$A \cdot B = (c_{ij})$, where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ik}b_{kj}$
Boolean conjunction	$A \wedge B = (a_{ij} \wedge b_{ij})$
Boolean disjunction	$A \vee B = (a_{ij} \vee b_{ij})$
Boolean product	$A \odot B = (c_{ij})$, where $c_{ij} = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee \cdots \vee (a_{ik} \wedge b_{kj})$

Some properties:

Commutative rule	$A + B = B + A$
Associative rules	$A + (B + C) = (A + B) + C$
	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$
Distributive rule	$A \cdot (B + C) = A \cdot B + A \cdot C$
Transpose and matrix operations	$(A \pm B)^t = A^t \pm B^t$
	$(A \cdot B)^t = B^t \cdot A^t$

Arithmetic

The following rules of arithmetic apply for all integers a, b and c.

- Commutative rules: $a + b = b + a$ $\quad a \cdot b = b \cdot a$
- Associative rules: $(a + b) + c = a + (b + c)$ $\quad (a \cdot b) \cdot c = a \cdot (b \cdot c)$
- Distributive rule: $a \cdot (b + c) = a \cdot b + a \cdot c$
- $(a \pm b)^2 = a^2 \pm 2ab + b^2$ $\quad (a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$
- $a^2 - b^2 = (a - b)(a + b)$ $\quad a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

The second-degree equation $ax^2 + bx + c = 0$, where $a \neq 0$ has a solution $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$.

Modular arithmetic

If $a_1 \cong b_1 (mod\ n)$ and $a_2 \cong b_2 (mod\ n)$, then	If $a \cong b (mod\ n)$ and $m \in \mathbb{Z}$, then
• $(a_1 + a_2) \cong (b_1 + b_2)(mod\ n)$. • $(a_1 - a_2) \cong (b_1 - b_2)(mod\ n)$. • $(a_1 \cdot a_2) \cong (b_1 \cdot b_2)(mod\ n)$.	• $m \cdot a \cong m \cdot b(mod\ n)$. • $a^m \cong b^m(mod\ n)$ for all integers $m \geq 0$.

Euler totient function

If $p_1, p_2, \ldots, p_k$ are the prime factors of n, then $\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right)$.

The Chinese remainder theorem:

If $m_i, \ldots, m_k$ are pairwise relatively prime integers, then the system of linear equations

$$\begin{cases} x \cong a_1\ (mod\ m_1) \\ x \cong a_2\ (mod\ m_2) \\ \vdots \\ x \cong a_k\ (mod\ m_k) \end{cases}$$

have a solution $x \cong \sum_{i=1}^{k} a_i y_i M_i\ (mod\ M)$, *where*

- $M = \prod_{i=1}^{k} m_i$,
- $M_i = \frac{M}{m_i}$ *and*
- $y_i \cong M_i^{-1}(mod\ m_i)$

Relation and functions

Let $R: A \to A$ be a relation on a set A. Then R is

- *reflexive* if $(x, x) \in R$ for all $x \in A$.
- *symmetric* if $(x, y) \in R \to (y, x) \in R$.
- *antisymmetric* if $[(x, y) \in R \wedge (y, x) \in R] \to x = y$.
- *transitive* if $[(x, y) \in R \wedge (y, z) \in R] \to (x, z) \in R$.

Let $f: A \to B$ be a function from a set A to a set B. Then f is

- *injective* (one-to-one) if $f(a_1) = f(a_2) \to a_1 = a_2$.
- *surjective* (onto) if $(\forall b \in B)(\exists a \in A)\ (f(a) = b)$.
- *bijective* (one-to-one correspondence) if f is both injective and surjective.