



UNIVERSITY
OF SKÖVDE

School of Engineering Science

WRITTEN EXAMINATION

Course: Engineering Materials and Applications

Sub-course

Course code: VP747A

Credits for written examination: 2 ECTS

Date 2026-06-04

Examination time 12:15 – 18:30

Examination responsible: Anna Syberfeldt

Teachers concerned: Masood Fathi

Aid at the exam/appendices: Nothing

Other

- Instructions
- ☐ Take a new sheet of paper for each teacher.
 - ☒ Take a new sheet of paper when starting a new question.
 - ☒ Write only on one side of the paper.
 - ☒ Write your name and personal ID No. on all pages you hand in.
 - ☒ Use page numbering.
 - ☒ Don't use a red pen.
 - ☒ Mark answered questions with a cross on the cover sheet.

Grade points (ECTS)

A 40 - 37 B 36 - 33 C 32 - 29 D 28 - 25 E 24 - 21 F 20 - 0

Examination results should be made public within 18 working days

Good luck!

Total number of pages: 3

Question 1: Dynamic Programming (10p)

A company has 4 marketing packages to distribute among 3 sales regions in order to maximize total profit. Each package must be assigned as a whole to one region, so a package cannot be split between regions. It is also possible to assign no package to one or more regions.

Because the selling conditions differ by region, profit depends on how many packages are assigned to that region. The manager wants to determine how many packages should be assigned to each region to maximize total profit.

The values in the table represent the total profit for each region, based on the number of packages assigned to it. For example, if 2 packages are assigned to Region 1, the total profit will be 12. The profit does not increase linearly, meaning that each additional package may contribute a different amount to the total profit.

Solve this problem using the *Dynamic Programming method*. What are the state, stage, and decision for this problem? Report the optimal number of packages to be assigned to each region to maximize total profit.

Use the *backward approach* and present all the calculations and solution steps. Follow the *tabular solution approach* and show each stage in a table.

Region	0	1	2	3	4
1	0	7	12	18	21
2	0	5	11	16	22
3	0	6	13	17	20

Question 2: Dual programming (5p)

Write the **dual LP** of the following primal LP model.

$$\text{Maximize } Z = 7x_1 + 3x_2 + 10x_3$$

Subject to:

$$2x_1 + x_2 + 4x_3 \leq 15$$

$$x_1 - 3x_2 + 2x_3 = 6$$

$$x_1, x_2, x_3 \geq 0$$

You can get help from the following table when writing the dual model.

Label	Primal Problem (or Dual Problem)	Dual Problem (or Primal Problem)
	Maximize Z (or W)	Minimize W (or Z)
Sensible	Constraint i :	Variable y_i (or x_i):
Odd	$\leq$ form $\leftarrow$	$y_i \geq 0$
Bizarre	$=$ form $\leftarrow$	Unconstrained
	$\geq$ form $\leftarrow$	$y_i \leq 0$
Sensible	Variable x_j (or y_j):	Constraint j :
Odd	$x_j \geq 0 \leftarrow$	$\geq$ form
Bizarre	Unconstrained $\leftarrow$	$=$ form
	$x_j' \leq 0 \leftarrow$	$\leq$ form

Question 3. Branch & Bound (8p)

A furniture company produces bookshelves and study desks, with production carried out across two workshops. Workshop 1 is responsible for cutting and frame preparation, while Workshop 2 handles finishing and final assembly. The company aims to determine the optimal number of each product to manufacture in the coming week. Each bookshelf generates a profit of \$80, whereas each study desk yields \$55. The available production capacity is limited to 14 hours in Workshop 1 and 26 hours in Workshop 2. In terms of processing requirements, each bookshelf consumes 3 hours in Workshop 1 and 4 hours in Workshop 2, while each study desk requires 2 hours in Workshop 1 and 5 hours in Workshop 2.

The LP model of the problem is as follows.

$$\text{Maximize } Z = 80x_1 + 55x_2$$

Subject to:

$$3x_1 + 2x_2 \leq 14$$

$$4x_1 + 5x_2 \leq 26$$

$$x_1, x_2 \geq 0 \text{ and integers}$$

Find the optimal production quantity for each product to maximize profit using the **Branch & Bound method**. *Present all the calculations and solution steps.* In addition, draw the complete Branch & Bound solution tree, clearly indicating the branching decisions, node solutions, objective values, and the reason for pruning each node.

Question 4. Simplex Method (A:8p; B:4P)

A) A workshop produces two products, P1 and P2, using three resources: cutting, assembly, and finishing. During the coming week, the workshop has 6 hours of cutting time, 10 hours of assembly time, and 18 hours of finishing time available. Each unit of P1 requires 1 hour of cutting and 3 hours of finishing, while each unit of P2 requires 2 hours of assembly and 2 hours of finishing. The profit earned is \$4 for each unit of P1 and \$6 for each unit of P2. The objective is to determine the optimal production quantities that maximize total profit.

Resources / Products	P1	P2	Available Capacity
Cutting	1	0	6
Assembly	0	2	10
Finishing	3	2	18
Profit per unit	4	6	-

The LP model of the problem is as follows.

Maximize $Z = 4x_1 + 6x_2$

Subject to:

$$x_1 \leq 6$$

$$2x_2 \leq 10$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

Solve the given LP problem using the **Simplex** algorithm. *Present all the calculations and solution steps. Follow the tabular solution approach and show all the steps in a table.*

B) Explain the fundamental differences between the **Simplex**, **Dual Simplex**, and **Big-M** methods. *Motivate your answer.*

Question 5: Solution methods (5P)

Discuss the key factors that determine whether an exact or an approximate method should be used for solving an industrial optimization problem. In your answer, explain the conditions under which each approach is preferred. *Motivate your answer.*