UNIVERSITY
OF SKÖVDE

School of Engineering Science

WRITTEN EXAMINATION

Course: Engineering Optimization: Methods and Applications

Sub-course

Course code: VP747A

Credits for written examination: 2 ECTS

Date 2025-08-20

Examination time 08:15 – 12:30

Examination responsible: Anna Syberfeldt

Teachers concerned: Masood Fathi

Aid at the exam/appendices: Nothing

Other

Instructions

- ☐ Take a new sheet of paper for each teacher.
- ☒ Take a new sheet of paper when starting a new question.
- ☒ Write only on one side of the paper.
- ☒ Write your name and personal ID No. on all pages you hand in.
- ☒ Use page numbering.
- ☒ Don't use a red pen.
- ☒ Mark answered questions with a cross on the cover sheet.

Grade points (ECTS)

A40 - 37 B 36 - 33 C 32 - 29 D 28 - 25 E 24 – 21 F 20 - 0

Examination results should be made public within 18 working days

Good luck!

Total number of pages: 3

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Question 1: Dynamic Programming (10p)

A company owns 3 manufacturing plants and has recruited 6 operators. The company manager wants to decide on the number of operators that should be assigned to each plant to maximize production. At least one operator should be assigned to each plant. The amount of production in each plant based on the number of operators is presented in the Table below.
Solve this problem using the **Dynamic Programming** method and find the optimal number of operators that should be assigned to each plant to maximize the total production. *Present all the calculations and solution steps.*
Use the ***backward approach*** *and present all the calculations and solution steps.*

Number of operators in a plant	Manufacturing plant		
	1	2	3
1	4	3	5
2	6	6	7
3	9	8	10
4	11	10	12

Additional help: Considering that at least one operator should be assigned to each plant, no more than 4 operators can be assigned to one specific plant.

Question 2: Dual programming (4p)

Write the **dual LP** of the following primal LP model.

$Maximize\ Z = 4x_1 + 2x_2 + 5x_3$
Subject to:
$x_1 + x_2 \geq 2$
$x_1 + 2x_2 + 3x_3 \leq 15$
$x_1, x_2, x_3 \geq 0$
You can get help from the following Table for writing the dual model.

Label	Primal Problem (or Dual Problem)	Dual Problem (or Primal Problem)
	Maximize Z (or W)	Minimize W (or Z)
	Constraint i:	Variable y_i (or x_i):
Sensible	$\leq$ form $\longleftrightarrow$	$y_i \geq 0$
Odd	$=$ form $\longleftrightarrow$	Unconstrained
Bizarre	$\geq$ form $\longleftrightarrow$	$y_i' \leq 0$
	Variable x_j (or y_j):	Constraint j:
Sensible	$x_j \geq 0$ $\longleftrightarrow$	$\geq$ form
Odd	Unconstrained $\longleftrightarrow$	$=$ form
Bizarre	$x_j' \leq 0$ $\longleftrightarrow$	$\leq$ form

Question 3: Simplex Method (A:8P; B:4P)

A) A company manufactures two products, X1 and X2, using three machines, A, B, and C. Machine A has 4 hours of capacity available during the coming week. The available capacity of machines B and C during the coming week is 12 hours and 18 hours, respectively. One unit of product X1 requires one hour of Machine A and 3 hours of Machine C. One unit of product X2 requires 2 hours of machines B and C. When one unit of X1 is sold in the market, it yields a profit of \$3, and that of X2 is \$5. The problem information is summarized in the Table below.

	Products		
Machines	**X1**	**X2**	**Available capacity in hours**
A	1	0	4
B	0	2	12
C	3	2	18
Profit per unit	\$3	\$5	-

The LP model of the problem to find the optimal product mix is given below.
Maximize Z= $3x_1+5x_2$
Subject to:
$x_1 \leq 4$
$2x_2 \leq 12$
$3x_1+2x_2 \leq 18$
$x_1, x_2 \geq 0$

Solve the given LP problem using the ***Simplex method****. Present all the calculations and solution steps.*

B) What are the main differences between the **Simplex, Dual Simplex,** and **Big-M** methods? Which one do you prefer to use to solve a linear programming model? Why? Motivate your answer.

Question 4: Branch & Bound (8P)

A glass production company produces windows and glass doors. Production occurs on two lines. Line 1 manufactures frames, while line 2 manufactures glass and assembles the final products. All of the company's products are manufactured with frames and glass. There are two upcoming products that the company wants to produce: Product 1, which is a glass door with a profit of \$ 70 per unit, and Product 2, which is a window with a profit of \$ 60 per unit. The number of hours of production time available for producing these new products is 12 hours in line 1 and 30 hours in line 2. Each unit of product A requires two hours in line 1, and six hours in line 2. Each unit of Product B requires three hours in line 1, and five hours in line 2. According to the information provided above, it is desired to determine which mix of the two products would be most profitable for the company.
The problem can be formulated as the LP model presented below.

Maximize profit = $70\ x_1 + 60\ x_2$
Subject to:
$2x_1 + 3x_2 \leq 12$
$6x_1 + 5x_2 \leq 30$
$x_1, x_2 \geq 0$

Decide on the best product mix to maximize the total profit using the ***Branch & Bound*** *method. Present all the calculations and solution steps.*
Note: Start branching from the variable with the larger decimal part.

Question 5: Solution methods (A:3P; B:3P)

A) What are the main advantages and disadvantages of exact and approximate solution methods?
B) Which solution method (exact or approximate) do you choose to solve an optimization problem? Why? Motivate your answer.