UNIVERSITY
OF SKÖVDE

School of Engineering Science

WRITTEN EXAMINATION

Course: Engineering Optimization: Methods and Applications

Sub-course

Course code: VP747A

Credits for written examination: 2 ECTS

Date 2025-05-28

Examination time 08:15 – 12:30

Examination responsible: Anna Syberfeldt

Teachers concerned: Masood Fathi

Aid at the exam/appendices: Nothing

Other

Instructions

- ☐ Take a new sheet of paper for each teacher.
- ☒ Take a new sheet of paper when starting a new question.
- ☒ Write only on one side of the paper.
- ☒ Write your name and personal ID No. on all pages you hand in.
- ☒ Use page numbering.
- ☒ Don't use a red pen.
- ☒ Mark answered questions with a cross on the cover sheet.

Grade points (ECTS)

A 40 - 37 B 36 - 33 C 32 - 29 D 28 - 25 E 24 – 21 F 20 - 0

Examination results should be made public within 18 working days

Good luck!

Total number of pages: 3

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Question 1: Dynamic Programming (10p)

A company has three unique manufacturing plants and has recently acquired five new machines. The company's manager is now tasked with determining the optimal number of machines that should be allocated to each plant in order to maximize production. The production levels of each plant are presented in the table below based on the number of machines assigned to them. Solve this problem using the ***Dynamic Programming*** *method.* What are the state, stage, and decision for this problem? Report the optimal number of machines at each plant and the maximum production.
Use the ***backward approach*** *and present all the calculations and solution steps.*

Number of Machines	Plant 1	Plan 2	Plant 3
0	0	0	0
1	50	20	45
2	70	45	70
3	80	75	90
4	100	110	105
5	130	150	120

Question 2: Dual programming (5p)

Write the **dual LP** of the following primal LP model.

$Maximize\ Z = 4x_1 + x_2 + 7x_3$
Subject to:
$x_1 + x_2 + x_3 = 10$
$5x_1 - x_2 + x_3 = 12$
$x_1 + x_2 - 3x_3 \leq 4$
$x_1, x_2, x_3 \geq 0$

You can get help from the following table when writing the dual model.

Label	Primal Problem (or Dual Problem)	Dual Problem (or Primal Problem)
	Maximize Z (or W)	Minimize W (or Z)
Sensible Odd Bizarre	Constraint i: $\leq$ form $\longleftrightarrow$ $=$ form $\longleftrightarrow$ $\geq$ form $\longleftrightarrow$	Variable y_i (or x_i): $\longleftrightarrow$ $y_i \geq 0$ $\longleftrightarrow$ Unconstrained $\longleftrightarrow$ $y_i' \leq 0$
Sensible Odd Bizarre	Variable x_j (or y_j): $x_j \geq 0$ $\longleftrightarrow$ Unconstrained $\longleftrightarrow$ $x_j' \leq 0$ $\longleftrightarrow$	Constraint j: $\longleftrightarrow$ $\geq$ form $\longleftrightarrow$ $=$ form $\longleftrightarrow$ $\leq$ form

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Question 3: Branch & Bound (8P)

A workshop has two types of machines (i.e., A and B). The company can manufacture two products, namely X and Y. All the products must visit both machines. The hours needed at each machine per product unit and the total available hours for each machine per week are given in the Table below. The profit of each product per unit sold for product X is 5$, and for product Y is 4$.

	Machine	
	A	B
Hours/week	5	45
Hours at each machine for product X	1	10
Hours at each machine for product Y	1	6

The LP model of the problem is as follows.

Maximize $Z= 5x+4y$

Subject to:

$x+y \leq 5$
$10x+6y \leq 45$
$x,y \geq 0$ and Integer

Find the optimal amount of production for each product to maximize the profit using the ***Branch & Bound*** *method. Present all the calculations and solution steps.*

Question 4: Simplex Method (A:8P; B:4P)

A) A company makes two products, *standard* and *deluxe*. A unit of the *standard* gives a profit contribution of 10$, while a unit of *deluxe* gives a profit contribution of 15$.
Two processes (i.e., grinding and polishing) are needed to produce the products. The company has a grinding capacity of 80 hours per week and a polishing capacity of 60 hours per week. The grinding and polishing times in hours for a unit of each type are given in the Table below. In addition, each unit of each product uses 4 kg of raw material, and the total available raw material at the company is 75 kg per week. The company wants to decide on a mix of products to maximize profit.

	Standard	**Deluxe**
Grinding	4	2
Polishing	2	5

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The LP model of the problem is as follows.

$$\begin{aligned} Max \quad & Z = 10\,x_1 + 15\,x_2 \\ \text{Subject to:} \quad & 4x_1 + 4x_2 \leq 75 \\ & 4x_1 + 2x_2 \leq 80 \\ & 2x_1 + 5x_2 \leq 60 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Solve the given LP problem using the ***Simplex*** algorithm. *Present all the calculations and solution steps.*

B) When do you use the ***Dual Simplex*** method to solve the linear programming model? Why? *Motivate your answer.*

Question 5: Solution methods (5P)

What factors affect your decision to choose exact or approximate solution methods when solving an industrial problem? Which method (exact or approximate) do you use when? *Motivate your answer.*