UNIVERSITY
OF SKÖVDE

School of Engineering Science

WRITTEN EXAMINATION

Course: Engineering Optimization: Methods and Applications

Sub-course

Course code: VP747A

Credits for written examination: 2 ECTS

Date 2024-08-29

Examination time 8:15 – 12:30

Examination responsible: Anna Syberfeldt

Teachers concerned: Masood Fathi

Aid at the exam/appendices: Nothing

Other

Instructions

- ☐ Take a new sheet of paper for each teacher.
- ☒ Take a new sheet of paper when starting a new question.
- ☐ Write only on one side of the paper.
- ☒ Write your name and personal ID No. on all pages you hand in.
- ☒ Use page numbering.
- ☒ Don't use a red pen.
- ☒ Mark answered questions with a cross on the cover sheet.

Grade points (ECTS)

A40 - 37 B 36 - 33 C 32 - 29 D 28 - 25 E 24 – 21 F 20 - 0

Examination results should be made public within 18 working days

Good luck!

Total number of pages: 3

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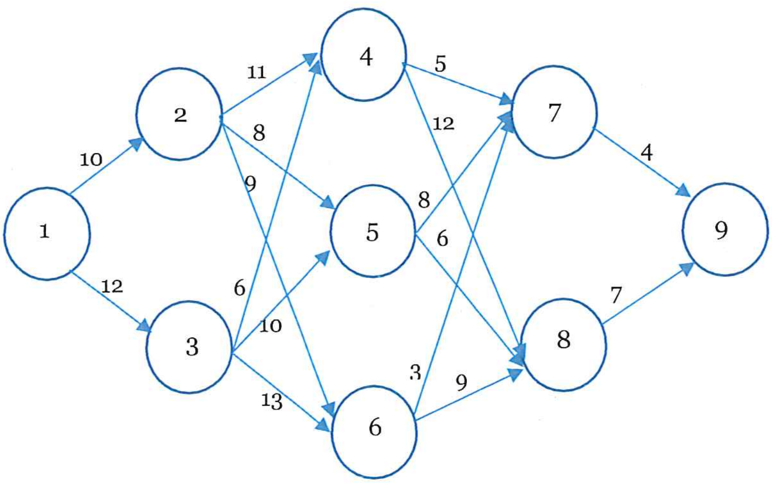
Question 1: Dynamic Programming (8p)

A manufacturing company needs to transport finished goods from its main production facility (Point 1) to a distribution center located in Point 9. The company has several warehouses and intermediate facilities where the goods can be stored temporarily before reaching the final destination. The network of possible routes between these points, along with the associated transportation costs, is provided in the diagram below.
Your task is to help the company find the most cost-effective route for transporting the goods from the production facility to the distribution center.
To solve this problem, use the **Dynamic Programming** method. Define the state, stage, and decision variables for this scenario. Determine and report the optimal route and the minimum transportation cost.
Use the ***backward approach*** *and present all the calculations and solution steps.*



Question 2: Dual programming (6p)

Write the **dual LP** of the following primal LP model.

$Maximize\ Z = 3x_1 + 7x_2 + 2x_3$

Subject to:
$2x_1 + x_2 + 4x_3 \leq 12$
$3x_1 + 4x_2 + x_3 = 15$
$x_1 - x_2 + x_3 \geq 5$
$x_1, x_2, x_3 \geq 0$

You can get help from the following Table for writing the dual model.

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Label	Primal Problem (or Dual Problem)	Dual Problem (or Primal Problem)
	Maximize Z (or W)	Minimize W (or Z)
	Constraint i:	Variable y_i (or x_i):
Sensible	$\leq$ form ⟷	$y_i \geq 0$
Odd	$=$ form ⟷	Unconstrained
Bizarre	$\geq$ form ⟷	$y_i' \leq 0$
	Variable x_j (or y_j):	Constraint j:
Sensible	$x_j \geq 0$ ⟷	$\geq$ form
Odd	Unconstrained ⟷	$=$ form
Bizarre	$x_j' \leq 0$ ⟷	$\leq$ form

Question 3: Simplex Method (A:8P; B:4P)

A) A small factory produces two types of products: Product A and Product B. The factory has the following resources available daily, Labor: 40 hours and Material: 60 units. The production requirements for each product are as follows: Product A requires 1 hour of labor and 3 units of material per unit, and Product B requires 2 hours of labor and 2 units of material per unit. The profit generated per unit of Product A is $30, and for Product B, it is $40.
Determine the number of units of Product A and Product B the factory should produce daily to maximize profit under the following assumption.

- The factory must use available resources optimally and cannot exceed the daily available labor and material.
- Both products can be produced in fractional units, though this might not be practical (purely for the mathematical model).

The linear programming model of the problem is as follow.

x_1: number of units of Product A produced.
x_2: number of units of Product B produced.

$Maximize\ Z = 30x_1 + 40x_2$

Subject to:
$1x_1 + 2x_2 \leq 40$ (Labor constraint)
$3x_1 + 2x_2 \leq 60$ (Material constraint)
$x_1, x_2 \geq 0$

Solve the given LP problem using the ***Simplex method***. *Present all the calculations and solution steps.*

B) What is the difference between **simplex**, **dual simplex**, and **Big-M** methods? When do you use each of these methods to solve a mathematical model? *Motivate your answer.*

Question 4: Branch & Bound (8P)

A manufacturing company is involved in the production of two products, x_1 and x_2. The production process for these products requires the use of two limited resources, R1 and R2. The objective of the company is to determine the optimal quantities of x_1 and x_2 that should be produced to maximize overall profit while adhering to the constraints imposed by the availability of resources. The profit generated per unit of each product, as well as the amount of each resource required for production, are detailed in the table below. The company has a total of 6 units of R1 and 154 units of R2 available for production. Furthermore, the production quantities must be integer values.

Product	Profit per unit ($)	Resource requirement R1	Resource requirement R2
x_1	7	1	18
x_2	11	1	34

The liner mathematical model of the problem is as follows.

$$Max \;\; Z = 7x_1 + 11x_2$$
$$x_1 + x_2 \leq 6$$
$$18x_1 + 34x_2 \leq 154$$
$$x_1 \geq 0 \, , \; x_2 \geq 0 \text{ \& Integer}$$

Find the optimal amount of production for each product to maximize the profit using the ***Branch & Bound*** *method. Present all the calculations and solution steps.*

Note: Start branching from the variable with the larger decimal part.

Question 5: Solution methods (A:3P; B:3P)

A) Discuss the main differences between exact and approximate solution methods. Under what conditions would you prefer one method over the other? Justify your choice with examples. *Motivate your answer.*

B) Explain the concept of "exploration vs. exploitation" in the context of optimization algorithms. How these two aspects are important in Genetic Algorithms and could guide the algorithm towards a good solution? *Motivate your answer.*