HÖGSKOLAN
I SKÖVDE

Institutionen för ingenjörsvetenskap

TENTAMEN

Kurs Diskret matematik (Discrete Mathematics)

Delkurs Salstentamen (Written-Exam)

Kurskod MA126 G1N Högskolepoäng för tentamen 5,0

Datum 2024-05-25 Skrivtid 09.15-14.30

Ansvarig lärare Yohannes Aklilu

Berörda lärare

Hjälpmedel/bilagor Studentens miniräknare, högskolans miniräknare

Övrigt

Anvisningar

- ☐ Ta nytt blad för varje lärare
- ☒ Ta nytt blad för varje ny fråga
- ☒ Skriv endast på en sida av papperet.
- ☒ Skriv namn och personnummer på samtliga inlämnade blad.
- ☒ Numrera lösbladen löpande.
- ☒ Använd inte röd penna.
- ☒ Markera med kryss på omslaget vilka uppgifter som är lösta.

Poänggränser

U-betyg(Fail grade): Not fulfilling the passing criterion.
G-betyg(Pass grade): At least 18 points.
VG-betyg (Pass with distinction grade): At least 28 points.

Skrivningsresultat bör offentliggöras inom 18 arbetsdagar

Lycka till!

Antal sidor totalt

Examination Instructions

The exam is assessed with a grade of *Pass with distinction* (VG), *Pass* (G) or *Fail* (U) based on how well your solutions demonstrate that the grading criteria for the course objectives have been met. Each task can give up to 6 points, a total of 36. For G-grade, a total of at least 18 points is required, for VG-grade at least 28.

To pass the examination with G-grade: Your answer to the tasks must be concise, but sufficiently detailed and formulated so that the line of thought can be easily followed. Some degree of calculation error can be acceptable, as long as the layout and motivation is correct. An answer, for example, with out any motivation, however, will not be accepted. Numerical values can be entered as expressions, suitably simplified, where root expressions, logarithms and exponential expressions can be included in addition to *pure numbers*, if needed.

In order to pass the exam with distinction (VG-grade): Your answer is required to be well-grounded and followed well-constructed mathematical reasoning that leads to a correct answer or conclusion. The answers must be well formulated and analysed, and draw relevant conclusions about the nature of the solutions in a logical and consistent manner.

The following course goals will be assessed on this examination:

- explain the central concepts and methods in discrete mathematics treated in the course,
- show good familiarity with integers and modular arithmetic, and in particular show some familiarity with the Euclidian algorithm, Fermat's little theorem, Euler's theorem and solve problems where they may be used,
- describe different graph theoretical relations, algorithms and their applications, e.g. graph searching, Kruskal's and Dijkstra's algorithms,
- analyze discrete mathematical structures and determine their characteristics using mathematical reasoning; typical examples of such structures are relations, graphs and trees,
- identify problems which can be solved by methods from discrete mathematics, and choose suitable methods and apply them in a structured way.

Good Luck!
YA

Examination tasks

Write your solutions on separate paper. Use new sheet for each task.

1. Let the universal set be the set of integers $\mathbb{Z}$. Given the set $A = \{x \in \mathbb{Z} : -10 \leq x \leq 10\}$ and the function $f : A \to \mathbb{Z}$ defined by $f(x) = x^2$.

   a) Is $f(x)$ a bijection (i.e. both 1-1 and onto)? Justify your answer. (2p)

   Let P be the set of prime numbers ≤ 100 and the set B be the image of A under f, that is

   $$B = \{y \in \mathbb{Z} : y = f(x) \text{ for } x \in A\}.$$

   b) How many elements does the set $P \cap B$ have? (2p)

   c) Give a complete list of the elements in the set $(A \setminus P) \cup B$. (2p)

2. a) Represent the following algebraic expressions with a binary tree. (2p)

   $$\left(\frac{3+\frac{1}{2}}{2-\frac{1}{3}}\right) + \left(\left(\frac{2}{5}+\frac{3}{7}\right) \cdot \frac{1}{4}\right)$$

   b) Determine the inverse of 12 in $\mathbb{Z}_{87}$, or justify why the inverse doesn't exist. (2p)

   c) Write the decimal number 133 in binary form. (2p)

3. For graph $G = (V, E)$ with nodes $V = \{a, b, c, d, e, f, g\}$ has the adjacency matrix A with respect to the natural ordering of the nodes.

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Use the adjacency matrix to motivate your answers to the following tasks (do not draw the graph).

a) Explain if this is a directed or undirected graph. (1p)

b) What is the sum of all the degrees of every vertex? (1p)

c) Is the graph connected? (2p)

d) Find the number of paths of length 4 between nodes a and c. (2p)

4. a) Let P, Q be logical propositions. Use the logical truth table to determine if the following logical expression is a tautology. (3p)

$$((P \vee Q) \wedge \neg P) \rightarrow Q$$

b) Let P, Q, R, S be logical propositions. If $P \wedge \neg Q$ is True and $\neg R \vee Q$ is False, determine the truth value of the proposition

$$(P \rightarrow Q) \rightarrow (R \rightarrow S).$$

(3p)

Hint: Rewrite and simplify the expression by using the property that $P \rightarrow Q$ is equivalent to $\neg P \vee Q$.

5.

a) Let R be a relation whose relation matrix is given by

$$M_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i. Is the relation reflexive, or anti-symmetric? Justify your answer. (2p)

ii. Determine the matrix representation for the relation $R \circ R$. (2p)

b) Construct two relations on the set $A = \{1, 2, 3, 4, 5\}$, one that is transitive and one that is not transitive. (2p)

6. a) Let us define the sum of positive divisors function

$$\sigma(n) = \sum_{d|n} d$$

in other words the sum of all positive divisors of a positive integer n. For example, $\sigma(6) = 1 + 2 + 3 + 6 = 24$ since the only positive divisors of 6 are $1, 2, 3, 6$ and we just add them all up. Prove that for any two prime numbers p and q such that $p \neq q$ we have that $\sigma(pq) = \sigma(p)\sigma(q)$. (3p)

b) Determine the shortest path from node A to G for the following weighted graph. (3p)