HÖGSKOLAN
I SKÖVDE

Institutionen för ingenjörsvetenskap

TENTAMEN

Kurs Diskret matematik (Discrete Mathematics)

Delkurs Salstentamen (Written-Exam)

Kurskod MA126 G1N

Högskolepoäng för tentamen 5,0

Datum 240319

Skrivtid 14.15-19.30

Ansvarig lärare Yohannes Aklilu

Berörda lärare

Hjälpmedel/bilagor Studentens miniräknare, högskolans miniräknare

Övrigt

Anvisningar

- ☐ Ta nytt blad för varje lärare
- ☒ Ta nytt blad för varje ny fråga
- ☒ Skriv endast på en sida av papperet.
- ☒ Skriv namn och personnummer på samtliga inlämnade blad.
- ☒ Numrera lösbladen löpande.
- ☒ Använd inte röd penna.
- ☒ Markera med kryss på omslaget vilka uppgifter som är lösta.

Poänggränser

U-betyg(Fail grade): Not fulfilling the passing criterion.
G-betyg(Pass grade): At least 18 points.
VG-betyg (Pass with distinction grade): At least 28 points.

Skrivningsresultat bör offentliggöras inom 18 arbetsdagar

Lycka till!

Antal sidor totalt

Examination Instructions

The exam is assessed with a grade of *Pass with distinction* (VG), *Pass* (G) or *Fail* (U) based on how well your solutions demonstrate that the grading criteria for the course objectives have been met. Each task can give up to 6 points, a total of 36. For G-grade, a total of at least 18 points is required, for VG-grade at least 28.

To pass the examination with G-grade: Your answer to the tasks must be concise, but sufficiently detailed and formulated so that the line of thought can be easily followed. Some degree of calculation error can be acceptable, as long as the layout and motivation is correct. An answer, for example, with out any motivation, however, will not be accepted. Numerical values can be entered as expressions, suitably simplified, where root expressions, logarithms and exponential expressions can be included in addition to *pure numbers*, if needed.

In order to pass the exam with distinction (VG-grade): Your answer is required to be well-grounded and followed well-constructed mathematical reasoning that leads to a correct answer or conclusion. The answers must be well formulated and analysed, and draw relevant conclusions about the nature of the solutions in a logical and consistent manner.

The following course goals will be assessed on this examination:

- explain the central concepts and methods in discrete mathematics treated in the course,
- show good familiarity with integers and modular arithmetic, and in particular show some familiarity with the Euclidian algorithm, Fermat's little theorem, Euler's theorem and solve problems where they may be used,
- describe different graph theoretical relations, algorithms and their applications, e.g. graph searching, Kruskal's and Dijkstra's algorithms,
- analyze discrete mathematical structures and determine their characteristics using mathematical reasoning; typical examples of such structures are relations, graphs and trees,
- identify problems which can be solved by methods from discrete mathematics, and choose suitable methods and apply them in a structured way.

Good Luck!
YA

Examination tasks

Write your solutions on separate paper. Use new sheet for each task.

1.

a) Let the universal $\mathbb{U}$ set be the set of all positive integers ≤ 30. Given the sets

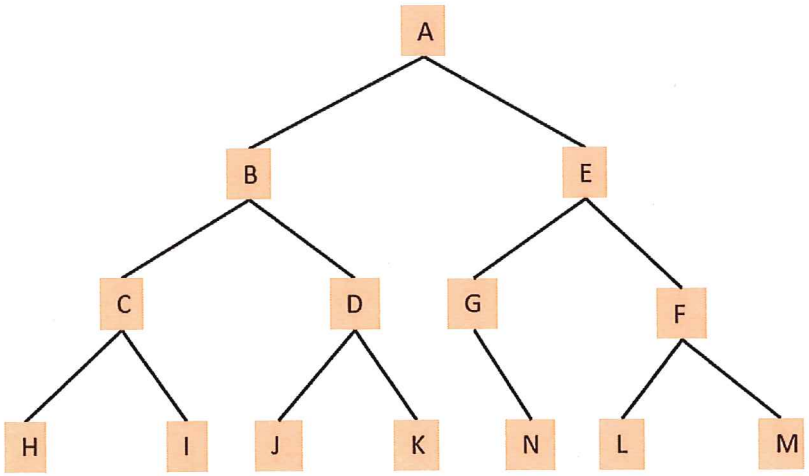
$$A_i = \{x \in \mathbb{U} \mid x \cong i \pmod 5\},$$

where $i = 0, 1, 2, 3, 4$.

i. Give a complete list of the elements in set A_2. (2p)

iii. Compute the intersection $A_1 \cap A_4$. (2p)

b) Use the rooted binary tree to determine which order the nodes are visited when applying post-order traversing. (2p)



Figur 1: Task 1b

2. a) Find all integer solutions to the linear Diophantine equation $28x + 44y = 36$. (3p)

b) Given $3a+5 \cong 1 \pmod{13}$ and $2b-6 \cong 10 \pmod{13}$. Find the smallest non-negative integer x such that $x \cong (4b + 2a^{-1}) \pmod{13}$. (3p)

3.

a) A simple undirected graph G has the nodes $V = \{a, b, c, d, e, f, g, h, i\}$ and edges

$$E = \{\{a, b\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, e\}, \{b, f\}, \{d, i\}, \{e, f\}, \{e, h\}, \{e, i\}, \{f, g\}, \{g, h\}, \{i, h\}\}$$

(4p)

i. Construct the adjacency matrix G_M of the graph.

ii. Does the graph has a Euler path? Justify your answer by using the adjacency matrix G_M.

b) Construct a connected undirected graph with 8 nodes where the degree of the nodes are 3, 2, 4, 2, 1, 4, 4 and 2. If it is not possible, justify why. (2p)

4. a) Let p, q and r be logical statements. Use the logical truth table for the following logical expression. (3p)

$$((\neg p \wedge r) \rightarrow q)) \vee (\neg r \rightarrow (p \vee \neg q)).$$

b) Let x, y and z be the logical statements. (3p)

- x: You get a passing grade on this course.
- y: You have studied hard.
- z: You have solved many exercises.

i. Describe the following statement using x, y, z and logical connectives.

You can pass this course, if you study hard and solve as many exercises as possible.

ii. Translate the expression into a statement:

$$(\neg y \wedge z) \Rightarrow \neg x.$$

5. a) Construct the matrix representation of a relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ that is symmetric, but not reflexive. (3p)

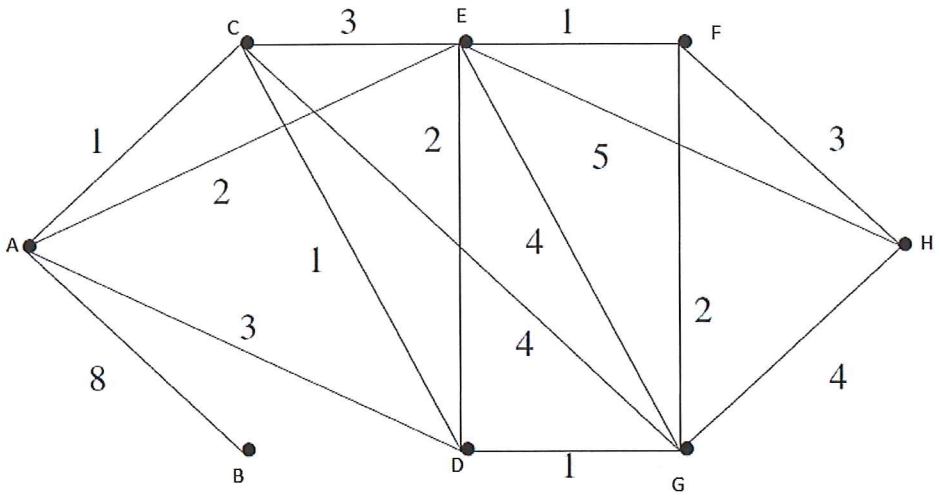
b) Let $B = \{1, 2, 3, 4, 5, 6, 7\}$ a set and

$$\mathcal{C} = \{\{2, 3, 5\}, \{1, 4\}, \{6\}, \{7\}\}$$

be partition of B. Define an equivalence relation on B whose equivalence classes are the partition $\mathcal{C}$. (3p)

6. a) Let a, b and c be positive integers. If a divides bc and $\gcd(a, c) = 1$, then prove that a divides the integer b. (3p)

b) Determine the minimal spanning tree for the graph. (3p)



Figur 2: Task 6b