

Institutionen för ingenjörsvetenskap

TENTAMEN

Kurs Diskret matematik (Discrete Mathematics)

Delkurs Salstentamen (Written-Exam)

Kurskod MA126 G1N

Högskolepoäng för tentamen 5,0

Datum 2026-05-13

Skrivtid 14.15-19.30

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Berörda lärare

Hjälpmedel/bilagor Studentens miniräknare, högskolans miniräknare, Befogat formelblad

Övrigt

Anvisningar

- Ta nytt blad för varje lärare
- Ta nytt blad för varje ny fråga
- Skriv endast på en sida av papperet.
- Skriv namn och personnummer på samtliga inlämnade blad.
- Numrera lösbladen löpande.
- Använd inte röd penna.
- Markera med kryss på omslaget vilka uppgifter som är lösta.

Poänggränser

U-betyg(Fail grade): Not fulfilling the passing criterion.

G-betyg(Pass grade): At least 18 points.

VG-betyg (Pass with distinction grade): At least 28 points.

Skrivningsresultat bör offentliggöras inom 18 arbetsdagar

Lycka till!

Antal sidor totalt

Examination Instructions

The exam is assessed with a grade of *Pass with distinction* (VG), *Pass* (G) or *Fail* (U) based on how well your solutions demonstrate that the grading criteria for the course objectives have been met. Each task can give up to 6 points, a total of 36. For G-grade, a total of at least 18 points is required, for VG-grade at least 28.

To pass the examination with G-grade: Your answer to the tasks must be concise, but sufficiently detailed and formulated so that the line of thought can be easily followed. Some degree of calculation error can be acceptable, as long as the layout and motivation is correct. An answer, for example, with out any motivation, however, will not be accepted. Numerical values can be entered as expressions, suitably simplified, where root expressions, logarithms and exponential expressions can be included in addition to *pure numbers*, if needed.

In order to pass the exam with distinction (VG-grade): Your answer is required to be well-grounded and followed well-constructed mathematical reasoning that leads to a correct answer or conclusion. The answers must be well formulated and analysed, and draw relevant conclusions about the nature of the solutions in a logical and consistent manner.

The following course goals will be assessed on this examination:

- explain the central concepts and methods in discrete mathematics treated in the course,
- show good familiarity with integers and modular arithmetic, and in particular show some familiarity with the Euclidian algorithm, Fermat's little theorem, Euler's theorem and solve problems where they may be used,
- describe different graph theoretical relations, algorithms and their applications, e.g. graph searching, Kruskal's and Dijkstra's algorithms,
- analyze discrete mathematical structures and determine their characteristics using mathematical reasoning; typical examples of such structures are relations, graphs and trees,
- identify problems which can be solved by methods from discrete mathematics, and choose suitable methods and apply them in a structured way.

Good Luck!

YA

Examination tasks

Write your solutions on separate paper. Use new sheet for each task.

1. (a) Let the universal \mathbb{U} set be the set of non-negative integers < 100 . Given the sets

$$A = \{x \in \mathbb{U} \mid x \cong 8 \pmod{16}\} \quad \text{and}$$

$$B = \{y \in \mathbb{U} \mid \text{the HEX representation of } y \text{ contains at least one C}\}.$$

- i. List all elements of B . (2p)
- ii. How many elements are there in the intersection $2^A \cap 2^B$? (2p)
- (b) Determine the expressions $A + B^t \cdot C$ and $2A - C^t$ that can be calculated. If no expression can be calculated, justify why. (2p)

The matrices are

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 0 & 1 \\ 1 & -3 & 0 \\ 0 & 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 1 & 0 & -2 \\ 0 & 2 & -1 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 2 & -1 \\ -1 & 0 & 5 \end{bmatrix}$$

2. (a) Let p, q and r be logical statements. Assume the the expressions $(\neg q \vee p) \wedge \neg r$ and $\neg p \wedge (r \vee \neg q)$ are both True. Find the truth value of the logical expressions: (3p)

$$((\neg p \wedge r) \vee q) \leftrightarrow (\neg q \rightarrow (\neg r \vee p)).$$

- (b) Let x, y and z be the logical statements. (3p)

- p : Grizzly bears have been seen in the area.
- q : Hiking is safe on the trail.
- r : Berries are ripe along the trail.

- i. Describe the following statement using p, q, r and logical connectives.

It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

- ii. Translate the expression into a statement:

$$r \rightarrow (q \leftrightarrow \neg p).$$

3. (a) The the integer x such that $0 \leq x < 21$ and $x \cong 2^{389} \pmod{21}$. (3p)
 (b) The congruence equation (3p)

$$\begin{cases} x \equiv 2 \pmod{3} \\ x \equiv 3 \pmod{4} \\ x \equiv 4 \pmod{7} \end{cases}$$

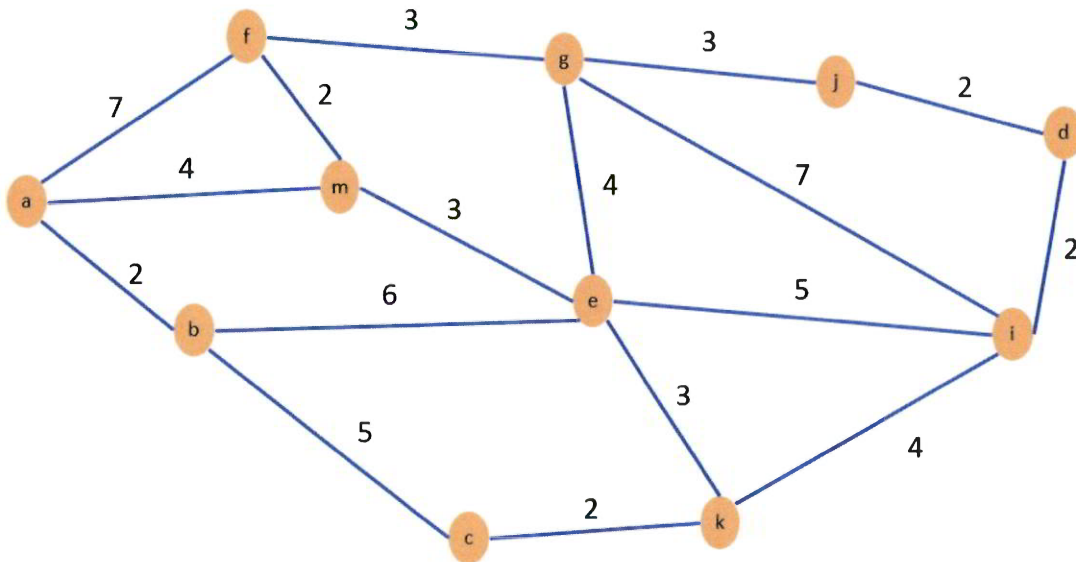
has a solution modulus m . Determine m and express all solutions of the congruence equation modulus m .

4. (a) Construct the Binary tree for the following expression given in post order: (2p)

$$4 \ 3 \ 0 + \ 2 \ 9 \ - \ + \ 600 \ 6 \ / \ *$$

- (b) Given the graph (2p/subtask)

- i. Find a Hamiltonian path for the underlying graph.
- ii. Determine the shortest path from node a to d.



Figur 1: Task 4b

5. (a) Construct a relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ that is transitive and symmetric, but not reflexive. (2p)
 (b) Construct the matrix representation of a relation S defined on the set $A = \{1, 2, 3, 4, 5\}$ that is symmetric and anti-symmetric, but not reflexive. (2p)

(c) Let R and S be relations whose relation matrices are defined by

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Determine the matrix representation for the composition relations $R \circ S$, R^2 and $S \circ R$, or justify why the operation cannot be formed. (2p)

6. (a) Define the Euler totient function (Euler ϕ -function). If p is a prime, and k is a positive integer, find a formula for $\phi(p^k)$. (3p)
- (b) A **binary search tree** is a searching algorithm that uses the principle that all the numbers (nodes) in the left subtree are smaller than the numbers in the right subtree, with the number in the root in between. The same property applies also to each subtree. Use this searching algorithm to construct a binary tree for the sequence

101 27 17 203 156 52 240.

(3p)

Discrete Mathematics, MA126G, 7,5hp
Formula Sheet

Logic

p	$\neg p$	p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
1	0	1	1	1	1	1	1
0	1	1	0	0	1	0	0
		0	1	0	1	1	0
		0	0	0	0	1	1

Some set theoretic and logical relations or properties

	Set theoretical rules	Logical Equivalences
Dubble Complement	$(A^c)^c = A^{cc} = A$	$\neg\neg p \Leftrightarrow p$
Complement/negation	$A \cap A^c = \emptyset$	$p \wedge \neg p \Leftrightarrow 0$
	$A \cup A^c = U$	$p \vee \neg p \Leftrightarrow 1$
Idempotent	$A \cap A = A$	$p \wedge p \Leftrightarrow p$
	$A \cup A = A$	$p \vee p \Leftrightarrow p$
Commutative laws	$A \cap B = B \cap A$	$p \wedge q \Leftrightarrow q \wedge p$
	$A \cup B = B \cup A$	$p \vee q \Leftrightarrow q \vee p$
Associative laws	$A \cap (B \cap C) = (A \cap B) \cap C$	$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$
	$A \cup (B \cup C) = (A \cup B) \cup C$	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$
Distributive laws	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
DeMorgan rules	$(A \cup B)^c = A^c \cap B^c$	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
	$(A \cap B)^c = A^c \cup B^c$	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
	$A \setminus B = A \cap B^c$	$p \rightarrow q \Leftrightarrow \neg p \vee q$

Definitions and some operations on logic and set theory

Power set	$2^A = \{B \mid B \subseteq A\}$	
Cartesian product	$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$	
Quantifiers	$\exists x$	There exists x
	$\forall x$	For all x
	$\neg(\forall x P(x)) \Leftrightarrow \exists x(\neg P(x))$	
	$\neg(\exists x P(x)) \Leftrightarrow \forall x(\neg P(x))$	

Matrices

Given matrices (or Boolean matrices) $A = (a_{ij})$ and $B = (b_{ij})$. We have the following operations as long as compatibility criterions are satisfied:

Matrix addition/subtraction	$A \pm B = (a_{ij} \pm b_{ij})$
Matrix multiplication	$A \cdot B = (c_{ij})$, where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{ik}b_{kj}$
Boolean conjunction	$A \wedge B = (a_{ij} \wedge b_{ij})$
Boolean disjunction	$A \vee B = (a_{ij} \vee b_{ij})$
Boolean product	$A \odot B = (c_{ij})$, where $c_{ij} = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee \dots \vee (a_{ik} \wedge b_{kj})$

Some properties:

Commutative rule	$A + B = B + A$
Associative rules	$A + (B + C) = (A + B) + C$
	$A \cdot (B \cdot C) = (A \cdot B) \cdot C$
Distributive rule	$A \cdot (B + C) = A \cdot B + A \cdot C$
Transpose and matrix operations	$(A \pm B)^t = A^t \pm B^t$
	$(A \cdot B)^t = B^t \cdot A^t$

Arithmetic

The following rules of arithmetic apply for all integers a, b and c .

- Commutative rules: $a + b = b + a$ $a \cdot b = b \cdot a$
- Associative rules: $(a + b) + c = a + (b + c)$ $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
- Distributive rule: $a \cdot (b + c) = a \cdot b + a \cdot c$
- $(a \pm b)^2 = a^2 \pm 2ab + b^2$ $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$
- $a^2 - b^2 = (a - b)(a + b)$ $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

The second-degree equation $ax^2 + bx + c = 0$, where $a \neq 0$ has a solution $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$.

Modular arithmetic

If $a_1 \cong b_1 \pmod{n}$ and $a_2 \cong b_2 \pmod{n}$, then <ul style="list-style-type: none"> • $(a_1 + a_2) \cong (b_1 + b_2) \pmod{n}$. • $(a_1 - a_2) \cong (b_1 - b_2) \pmod{n}$. • $(a_1 \cdot a_2) \cong (b_1 \cdot b_2) \pmod{n}$. 	If $a \cong b \pmod{n}$ and $m \in \mathbb{Z}$, then <ul style="list-style-type: none"> • $m \cdot a \cong m \cdot b \pmod{n}$. • $a^m \cong b^m \pmod{n}$ for all integers $m \geq 0$.
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Euler totient function

If p_1, p_2, \dots, p_k are the prime factors of n , then $\phi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_k}\right)$.

The Chinese remainder theorem:

If m_1, \dots, m_k are pairwise relatively prime integers, then the system of linear equations

$$\begin{cases} x \cong a_1 \pmod{m_1} \\ x \cong a_2 \pmod{m_2} \\ \vdots \\ x \cong a_k \pmod{m_k} \end{cases}$$

have a solution $x \cong \sum_{i=1}^k a_i y_i M_i \pmod{M}$, where

- $M = \prod_{i=1}^k m_i$,
- $M_i = \frac{M}{m_i}$ and
- $y_i \cong M_i^{-1} \pmod{m_i}$

Relation and functions

Let $R: A \rightarrow A$ be a relation on a set A . Then R is

- reflexive if $(x, x) \in R$ for all $x \in A$.
- symmetric if $(x, y) \in R \rightarrow (y, x) \in R$.
- antisymmetric if $[(x, y) \in R \wedge (y, x) \in R] \rightarrow x = y$.
- transitive if $[(x, y) \in R \wedge (y, z) \in R] \rightarrow (x, z) \in R$.

Let $f: A \rightarrow B$ be a function from a set A to a set B . Then f is

- injective (one-to-one) if $f(a_1) = f(a_2) \rightarrow a_1 = a_2$.
- surjective (onto) if $(\forall b \in B)(\exists a \in A)(f(a) = b)$.
- bijective (one-to-one correspondence) if f is both injective and surjective.